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|  | Lecture Notes | 9th May 2023 |

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|  | Subjects to Cover |  |
|  |  |  |
| 1 | ODE of the first order | First Midterm |
| 2 | ODE of higher orders |
|  |  |  |
| 3 | Power Series Solutions | Second Midterm |
| 4 | Laplace Transforms |
|  |  |  |
| 5 | Quantitative Methods: Numerical Solution | If time permits |
|  |  |  |
| 6 | Fourier Series, Fourier Integral, Fourier Transform | Everything for Finals |
| 7 | Sturm-Liouville problem |
| 8 | Partial Differential Equations (PDE) |
| 9 | Diffusion (heat) PDE |
| 10 | Wave PDE |
| 11 | Laplace PDE |

**Ordinary Differential Equations (ODE)**:

A differential equation is an equation containing a function like y(t) and its derivatives y’, y’’, y’’’, …

It is called ordinary if the dependent function y has only 1 independent variable.

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Example 1:

Highest degree = 2, therefore Second Order ODE

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Example 2: Electrical Engineering (RLC Circuit)

L, R, C = constants

I, V are functions of Time t

I(t) = current at time t

V(t) = voltage at time t

Second Order, homogenous if V’ = 0, also linear

Differential equation = f(t)

If f(t) = 0 ==> homogeneous

If f(t) ≠ 0 ==> non-homogeneous

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Example 3: Mechanical Engineering (Mechanical Vibrations)

m,c,k = constants

Second order,

If f(t) = 0 ==> homogeneous

If f(t) ≠ 0 ==> non-homogeneous

Linear

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Example 4: Civil Engineering (Elastic beam)

(4) ==> Fourth Derivative

Fourth Order

Linear

Homogeneous

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Example 5: Industrial Engineering (Speed control of a DC motor)

a, b = constant

First order

Linear

Homogeneous

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**First Order Differential Equation**:

* Separation of variables
* Integration Factor
* Variation of parameters
* Exact Equations

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**Separation of Variables**:

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**Solve the following ODE using separation of variables**:

Example 1:

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Example 2:

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Example 3:

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… (By Partial Fraction)

**Integration factor:**

… (1)

Multiplying equation (1) by the Integration Factor we get,

By Product Rule of Differentiation, the LHS becomes

Integrating both sides

**Solve the following ODE using Integration Factor:**

Example 1:

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Multiplying both sides of the given differential equation by the integration factor we get,

By Product rule we can simplify the LHS as,

Integrating both sides we get,

Example 2:

**-----------------------------------------------------------------------------------------------------------------------------**

Multiplying both sides of the given differential equation by the integration factor we get,

By Product rule we can simplify the LHS as,

Integrating both sides we get,

Example 3:

**-----------------------------------------------------------------------------------------------------------------------------**

Multiplying both sides of the given differential equation by the integration factor we get,

By Product rule we can simplify the LHS as,

Integrating both sides we get,

… (i)

Let

Substituting in (i) we get,

… (ii)

Resubstituting in (ii)